

# Appendix Tables

**Table 1.** Test problems for the benchmark

	<b>Problem</b>	<b>N</b>	<b>Bounds</b>	<b>Solution</b>
1	Ackley	4	$[-32, 32] \times 4$	0.000
2	Amgm	5	$[0, 10] \times 5$	0.000
3	Bohachevsky	5	$[-10, 10] \times 5$	0.000
4	Brent	2	$[-100, 100] \times 2$	0.000
5	Brown	6	$[-1, 4] \times 6$	0.000
6	Bukin 2	2	$[[ -15, -5], [-3, 3]]$	-124.750
7	Bukin 4	2	$[[ -15, -5], [-3, 3]]$	0.000
8	Cigar	2	$[-100, 100] \times 2$	0.000
9	Deckkers-Aarts	2	$[-20, 20] \times 2$	-24777.000
10	Dixon & Price	5	$[-10, 10] \times 5$	0.000
11	Biggs EXP2	2	$[0, 20] \times 2$	0.000
12	Goldstein Price	2	$[-6, 6] \times 2$	3.000
13	Griewank	5	$[-600, 600] \times 5$	0.000
14	Katsuura	2	$[0, 100] \times 2$	1.000
15	Levy 5	2	$[-2, 2] \times 2$	-176.138
16	Michalewicz	2	$[0, 3.1415] \times 2$	-1.801
17	Mishra 7	2	$[-10, 10] \times 2$	0.000
18	Multimodal	4	$[-10, 10] \times 4$	0.000
19	Perm	4	$[-4, 4] \times 4$	0.000
20	Perm 2	2	$[-2, 2] \times 2$	0.000
21	Rastrigin	2	$[-5.12, 5.12] \times 2$	0.000
22	Rosenbrock	2	$[-2.048, 2.048] \times 2$	0.000
23	Salomon	2	$[-50, 50] \times 2$	0.000
24	Sargan	6	$[-100, 100] \times 6$	0.000
25	Sum of Different Powers	5	$[-1, 1] \times 5$	0.000
26	Step	6	$[-100, 100] \times 6$	0.000
27	Styblinski-Tang	2	$[-5, 5] \times 2$	-78.332
28	Trid	2	$[-4, 4] \times 2$	-2.000
29	Weierstrass	5	$[-0.5, 0.5] \times 5$	0.000
30	Zakharov	10	$[-5, 10] \times 10$	0.000

**Table 2.** Benchmark results

Problem	N	Budget	Reached objective value			
			pSeven	pSeven DSBO	RBFOpt	Bayesian
Ackley	4	5N	<u>19.52</u>	19.98	19.94	19.97
		10N	18.57	16.88	18.70	<u>8.29</u>
		15N	12.85	5.36	18.30	<u>4.17</u>
Amgm	5	5N	0.00	0.00	0.00	<u>0.00</u>
		10N	0.00	0.00	0.00	<u>0.00</u>
		15N	0.00	0.00	0.00	<u>0.00</u>
Bohachevsky	5	5N	6.62	<u>2.79</u>	3.32	19.29
		10N	0.02	<u>0.02</u>	1.44	7.05
		15N	0.05	<u>0.05</u>	1.12	3.04
Brent	2	5N	8.45	68.44	<u>5.62</u>	25.88
		10N	0.00	<u>0.00</u>	1.41	0.02
		15N	<u>0.00</u>	0.00	0.00	0.01
Brown	6	5N	<u>0.70</u>	<u>0.70</u>	1.66	6.12
		10N	0.83	0.80	<u>0.00</u>	47.25
		15N	1.45	1.48	<u>0.00</u>	7.94
Bukin 2	2	5N	-124.75	<u>-124.75</u>	-63.85	16.46
		10N	-94.21	<u>-124.75</u>	-124.75	-28.81
		15N	<u>-124.75</u>	-124.75	-124.75	-91.89
Bukin 4	2	5N	<u>0.00</u>	<u>0.00</u>	1.94	0.18
		10N	0.02	<u>0.00</u>	0.03	0.01
		15N	0.00	<u>0.00</u>	0.01	0.01
Cigar	2	5N	69.13	<u>0.00</u>	91751067.72	659554.03
		10N	<u>0.00</u>	0.00	33108.72	519.02
		15N	0.00	<u>0.00</u>	702.90	9986.71
Deckkers-Aarts	2	5N	-608.19	-607.60	<u>-24328.34</u>	-2800.62
		10N	-19505.00	<u>-24710.29</u>	-24541.40	-632.66
		15N	-24744.68	<u>-24776.48</u>	-24666.45	-2825.37

Dixon & Price	5	5N	<u>45.10</u>	53.51	99.49	457.65
		10N	5.40	<u>5.39</u>	9.07	35.21
		15N	54.48	54.53	<u>2.09</u>	61.82
Biggs EXP2	2	5N	0.94	1.34	<u>0.76</u>	1.94
		10N	0.44	<u>0.08</u>	0.76	1.24
		15N	0.39	0.07	0.73	<u>0.05</u>
Goldstein Price	2	5N	<u>53.68</u>	57.00	205817.74	879.19
		10N	809.55	809.55	<u>46.74</u>	198.43
		15N	<u>12.67</u>	93.71	32.32	976.97
Griewank	5	5N	1.70	<u>1.05</u>	2.01	6.51
		10N	1.00	<u>0.02</u>	0.89	1.75
		15N	0.96	<u>0.05</u>	0.67	1.53
Katsuura	2	5N	<u>1.00</u>	<u>1.00</u>	11.93	<u>1.00</u>
		10N	<u>1.00</u>	296.20	2.29	<u>1.00</u>
		15N	<u>1.00</u>	<u>1.00</u>	<u>1.00</u>	<u>1.00</u>
Levy 5	2	5N	<u>-155.26</u>	<u>-155.26</u>	14.64	-13.35
		10N	<u>-58.30</u>	<u>-58.30</u>	-34.82	-36.14
		15N	<u>-44.43</u>	-33.55	-35.26	-36.12
Michalewicz	2	5N	<u>-1.00</u>	<u>-1.00</u>	-0.41	-0.71
		10N	-1.79	-1.79	-1.16	<u>-1.79</u>
		15N	<u>-1.56</u>	-0.74	-1.21	-0.79
Mishra 7	2	5N	1.74	3.23	0.00	<u>0.00</u>
		10N	0.07	0.07	0.44	<u>0.00</u>
		15N	0.03	0.01	0.03	<u>0.00</u>
Multimodal	4	5N	<u>0.70</u>	0.71	6.22	8.24
		10N	0.20	0.21	<u>0.14</u>	0.16
		15N	0.02	0.02	<u>0.00</u>	0.07
Perm	4	5N	434.64	434.64	11481.35	<u>153.56</u>
		10N	294.84	295.04	403.86	<u>261.06</u>
		15N	11.11	<u>2.90</u>	37.95	375.75

Perm 2	2	5N	139.95	67.87	394.87	<b><u>1.99</u></b>
		10N	10.92	1.18	0.24	<b><u>0.07</u></b>
		15N	1.62	<b><u>0.00</u></b>	0.02	1.98
Rastrigin	2	5N	<b><u>0.03</u></b>	<b><u>0.03</u></b>	6.38	5.69
		10N	7.45	7.45	<b><u>4.07</u></b>	18.47
		15N	<b><u>1.43</u></b>	1.43	3.98	14.70
Rosenbrock	2	5N	0.99	0.99	4.18	<b><u>0.32</u></b>
		10N	0.23	<b><u>0.13</u></b>	2.25	1.50
		15N	0.07	<b><u>0.00</u></b>	2.02	1.40
Salomon	2	5N	0.30	<b><u>0.20</u></b>	1.24	1.93
		10N	<b><u>0.21</u></b>	<b><u>0.21</u></b>	0.92	0.98
		15N	<b><u>0.35</u></b>	0.58	0.90	0.91
Sargan	6	5N	1353.38	<b><u>1198.12</u></b>	1412.71	21535.02
		10N	0.00	<b><u>0.00</u></b>	21.15	5294.26
		15N	0.18	<b><u>0.00</u></b>	8.25	1264.20
Sum of Different Powers	5	5N	0.00	0.01	<b><u>0.00</u></b>	0.01
		10N	0.00	<b><u>0.00</u></b>	0.00	0.00
		15N	<b><u>0.00</u></b>	0.00	0.00	0.00
Step	6	5N	376.00	<b><u>16.00</u></b>	110.00	1990.00
		10N	<b><u>0.00</u></b>	<b><u>0.00</u></b>	7.00	665.00
		15N	<b><u>0.00</u></b>	<b><u>0.00</u></b>	2.00	257.00
Styblinski-Tang	2	5N	<b><u>-53.83</u></b>	<b><u>-53.83</u></b>	-43.29	-30.42
		10N	-51.88	<b><u>-75.59</u></b>	-56.36	-50.06
		15N	-64.19	<b><u>-78.33</u></b>	-76.87	-52.64
Trid	2	5N	-1.99	<b><u>-1.99</u></b>	-0.74	-1.64
		10N	<b><u>-2.00</u></b>	-2.00	-2.00	-2.00
		15N	<b><u>-2.00</u></b>	-2.00	-2.00	-2.00
Weierstrass	5	5N	4.21	<b><u>4.18</u></b>	6.11	5.62
		10N	2.68	2.59	3.01	<b><u>2.41</u></b>
		15N	3.49	3.55	2.66	<b><u>2.02</u></b>

Zakharov	10	5N	206.09	206.09	408.49	<b><u>136.54</u></b>
		10N	208.58	209.44	370.06	<b><u>101.97</u></b>
		15N	176.47	173.54	254.48	<b><u>159.59</u></b>